The Query Containment Problem: Set Semantics *vs*. Bag Semantics

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PROBLEMS

Problems worthy
of attack
prove their worth
by hitting back.

in: *Grooks* by Piet Hein (1905-1996)

An Old Problem in Database Theory

- Database theory research has been going on for more than four decades.
- Over the years, it has had numerous successes.
- Yet, in spite of concerted attacks, some problems have been "hitting back" and resisting solution.

- This talk is about the

  conjunctive query containment problem under bag semantics,

  an old, but persistent problem that remains open to date.
- This problem was introduced exactly 20 years ago by Surajit Chaudhuri and Moshe Y. Vardi.
- This talk is dedicated to them.

Outline of the Talk

- Background and motivation
- Query containment under set semantics
- Query containment under bag semantics
  - Problem description
  - Partial progress to date
- Concluding remarks and outlook.

The Query Containment Problem

Let Q_1 and Q_2 be two database queries.

- $Q_1 \subseteq Q_2$ means that for every database D, we have that $Q_1(D) \subseteq Q_2(D)$, where $Q_i(D)$ is the set of all tuples returned by evaluating Q_i on D.
- The Query Containment Problem asks: given two queries Q_1 and Q_2, is $Q_1 \subseteq Q_2$?
- For boolean queries ("**true**" or "**false**), query containment amounts to logical implication $Q_1 \models Q_2$, which is a fundamental problem in logic.

The Query Containment Problem

- Encountered in several different areas, including
  - Query processing

    query equivalence reduces to query containment:

    $Q_1 \equiv Q_2$ if and only if $Q_1 \subseteq Q_2$ and $Q_2 \subseteq Q_1$.

  - Decision-support
    - Q_1 may be much easier to evaluate than Q_2.
    - If $Q_1 \subseteq Q_2$, then

      Q_1 provides a sound approximation to Q_2.

- Tight connections with constraint satisfaction (but this is another talk).

Complexity of Query Containment

The Query Containment Problem:
Given queries Q_1, Q_2, is $Q_1 \subseteq Q_2$?
In other words:
Is $Q_1(D)$ contained in $Q_2(D)$, for **all** databases D?

Note: Can't just try every database D – **infinitely** many!

Trakhtenbrot's Theorem (1949):
The set of finitely valid first-order sentences is undecidable.

Corollary: For first-order queries, the query containment problem is undecidable.

Conjunctive Queries and their Extensions

Extensive study of the query containment problem
for conjunctive queries and their extensions.

- Conjunctive queries: the most frequently asked queries
  They are the SELECT-PROJECT-JOIN queries.

- Unions of conjunctive queries.

- Conjunctive queries with inequalities $\neq$ and arithmetic comparisons $\leq$ and $\geq$.

Conjunctive Queries and Their Extensions

- Conjunctive Query:
  - $Q(x_1,\ldots,x_k)$: $\exists z_1 \ldots \exists z_m\, \varphi(x_1,\ldots,x_k,z_1,..z_m)$,
    where φ is a conjunction of atoms.
  - Example:
    TAUGHT-BY(x,y): $\exists$ z(ENROLLS(x,z) $\wedge$ TEACHES(y,z))
    Written as a logic rule:
    TAUGHT-BY(x,y):- ENROLLS(x,z), TEACHES(y,z)
- Union of Conjunctive Queries
  - Example: Path of length at most 2:
    Q(x,y): E(x,y) $\vee$ $\exists$ z(E(x,z) $\wedge$ E(z,y))
- Conjunctive Query with $\neq$
  - Example: At least two different paths of length 2:

Q(x,y): $\exists$ z $\exists$ w(E(x,z) $\wedge$ E(z,y) $\wedge$ E(x,w) $\wedge$ E(w,y) $\wedge$ z $\neq$ w).

Complexity of Conjunctive Query Containment

- **Theorem:** Chandra and Merlin – 1977
  For conjunctive queries, the containment problem is NP-complete.

- **Note:**
  - NP-hardness: reduction from 3-Colorability
  - Membership in NP is not obvious.
    It is a consequence of the following result.

Complexity of Conjunctive Query Containment

Theorem: Chandra and Merlin – 1977

For Boolean conjunctive queries Q_1 and Q_2, the following are equivalent:

- $Q_1 \subseteq Q_2$.
- There is a homomorphism $h : D[Q_2] \rightarrow D[Q_1]$, where $D[Q_i]$ is the canonical database of Q_i.

Example: Conjunctive query and canonical database

- Q:- E(x,y), E(y,z), E(z,x)
- D[Q] = { E(X,Y), E(Y,Z), E(Z,Y) }

Unions of Conjunctive Queries

Theorem: Sagiv & Yannakakis - 1980

The query containment problem for unions of conjunctive queries is NP-complete.

Note:

- Clearly, this problem is NP-hard, since it is at least as hard as conjunctive query containment.
- Membership in NP is **not** obvious.
  - It is a consequence of the following result.

Unions of Conjunctive Queries

Theorem: Sagiv & Yannakakis - 1980

For all conjunctive queries Q_1, …, Q_n, $Q'_1,\ldots,Q'_m$, the following two statements are equivalent:

- $Q_1 \cup \ldots \cup Q_n \subseteq Q'_1 \cup \ldots \cup Q'_m$.
- For every $i \leq n$, there is $j \leq m$, such that $Q_i \subseteq Q'_j$.

Note:

- The proof uses the Chandra-Merlin Theorem.
- For membership in NP:
  - we first guess n pairs (Q'_{k_i}, h_{k_i}); then
  - we verify that for every $i \leq n$, the function h_{k_i} is a homomorphism from $D[Q'_{k_i}]$ to $D[Q_i]$.

Conjunctive Queries with Arith. Comparisons

Theorem: The query containment problem for conjunctive queries with $\neq$, $\leq$, $\geq$ is Π_2^p-complete.

- Klug – 1988: Membership in Π_2^p.
  Suffices to test containment on exponentially many "canonical" databases.
- van der Meyden – 1992:
  Π_2^p-hardness, even for conjunctive queries with only $\neq$.

The Complexity Class Π_2^p

- Π_2^p is a complexity class that is sandwiched between NP and PSPACE, i.e.,

  $$NP \subseteq \Pi_2^p \subseteq PSPACE.$$

- The prototypical Π_2^p -complete problem is $\forall\exists$SAT, i.e., the restriction of QBF to formulas of the form

  $\forall x_1 \ldots \forall x_m \exists y_1 \ldots \exists y_n \varphi$.

Complexity of Query Containment

Class of Queries	**Complexity of Query Containment**
Conjunctive Queries	NP-complete Chandra & Merlin – 1977
Unions of Conjunctive Queries	NP-complete Sagiv & Yannakakis - 1980
Conjunctive Queries with $\neq$, $\leq$, $\geq$	Π_2^p-complete Klug 1988, van der Meyden -1992
First-Order (SQL) queries	Undecidable Trakhtenbrot - 1949

Complexity of Query Containment

- So, the complexity of query containment for conjunctive queries and their variants is well understood.

  Caveat:

- All preceding results assume **set semantics**, i.e., queries take **sets** as inputs and return **sets** as output (duplicates are eliminated).

- DBMS, however, use **bag semantics**, since they return **bags** (duplicates are **not** eliminated).

A *Real* Conjunctive Query

- Consider the following SQL query:

  Table `Employee` has attributes `salary, dept, …`

```
SELECT salary
FROM   Employee
WHERE  dept = 'CS'
```

- SQL keeps duplicates, because:
  - Duplicates are important for aggregate queries.
  - In general, bags can be more "efficient" than sets.

Query Evaluation under Bag Semantics

Operation	Multiplicity
Union $R_1 \cup R_2$	$m_1 + m_2$
Intersection $R_1 \cap R_2$	$\min(m_1, m_2)$
Product $R_1 \times R_2$	$m_1 \times m_2$
Projection and Selection	Duplicates are not eliminated

- R_1

A	B
1	2
1	2
2	3

- R_2

B	C
2	4
2	5

- $(R_1 \bowtie R_2)$

A	B	C
1	2	4
1	2	4
1	2	5
1	2	5

Bag Semantics

Chaudhuri & Vardi – 1993

Optimization of ***Real*** Conjunctive Queries

- Called for a re-examination of conjunctive-query optimization under bag semantics.
- In particular, they initiated the study of the containment problem for conjunctive queries containment under bag semantics.

Bag Semantics vs. Set Semantics

- For bags R_1, R_2:
  $R_1 \subseteq_{BAG} R_2$ if $m(\mathbf{a},R_1) \leq m(\mathbf{a},R_2)$, for every tuple **a**.
- $Q^{BAG}(D)$: Result of evaluating Q on (bag) database D.
- $Q_1 \subseteq_{BAG} Q_2$ if for every (bag) database D, we have that

$$Q_1^{BAG}(D) \subseteq_{BAG} Q_2^{BAG}(D).$$

Fact:

- $Q_1 \subseteq_{BAG} Q_2$ implies $Q_1 \subseteq Q_2$.
- The converse does **not** always hold.

Bag Semantics vs. Set Semantics

Fact: $Q_1 \subseteq Q_2$ does not imply that $Q_1 \subseteq_{BAG} Q_2$.

Example:

- $Q_1(x)$:- $P(x), T(x)$
- $Q_2(x)$:- $P(x)$

- $Q_1 \subseteq Q_2$ (obvious from the definitions)
- $Q_1 \not\subseteq_{BAG} Q_2$
- Consider the (bag) instance D = {P(a), T(a), T(a)}. Then:
  - $Q_1(D) = \{a,a\}$
  - $Q_2(D) = \{a\}$, so $Q_1(D) \not\subseteq Q_2(D)$.

Query Containment under Bag Semantics

- Chaudhuri & Vardi - 1993 stated that:

  Under bag semantics, the containment problem for conjunctive queries is Π_2^p-hard.

- **Problem:**
  - What is the **exact complexity** of the containment problem for conjunctive queries under bag semantics?
  - Is this problem **decidable**?

Query Containment Under Bag Semantics

- 20 years have passed since the containment problem for conjunctive queries under bag semantics was raised.
- Several attacks to solve this problem have failed.
- At least two flawed PhD theses on this problem have been produced.
- No proof of the claimed Π_2^p-hardness of this problem has been provided.

Query Containment Under Bag Semantics

- The containment problem for conjunctive queries under bag semantics remains **open** to date.

- However, progress has been made towards the containment problem under bag semantics for the two main extensions of conjunctive queries:
  - Unions of conjunctive queries
  - Conjunctive queries with ≠

Unions of Conjunctive Queries

Theorem: Ioannidis & Ramakrishnan – 1995
Under bag semantics, the containment problem for unions of conjunctive queries is **undecidable**.

Hint of Proof:
Reduction from Hilbert's 10th Problem.

Hilbert's 10th Problem

- Hilbert's 10th Problem – 1900

  (10th in Hilbert's list of 23 problems)

  Find an algorithm for the following problem:

  Given a polynomial $P(x_1,\dots,x_n)$ with integer coefficients, does it have an all-integer solution?

- Matiyasevich – 1971
  - Hilbert's 10th Problem is **undecidable**, hence **no** such algorithm exists.

Hilbert's 10^{th} Problem

- **Fact:** The following variant of Hilbert's 10^{th} Problem is **undecidable**:
  - Given two polynomials $p_1(x_1,\ldots x_n)$ and $p_2(x_1,\ldots x_n)$ with positive integer coefficients and no constant terms, is it true that $p_1 \leq p_2$?

    In other words, is it true that $p_1(a_1,\ldots,a_n) \leq p_2(a_1,\ldots a_n)$, for all positive integers $a_1,\ldots,a_n$?

- Thus, there is no algorithm for deciding questions like:
  - Is $3x_1^4x_2x_3 + 2x_2x_3 \leq x_1^6 + 5x_2x_3$?

Unions of Conjunctive Queries

Theorem: Ioannidis & Ramakrishnan – 1995

Under bag semantics, the containment problem for unions of conjunctive queries is **undecidable**.

Hint of Proof:

- Reduction from the previous variant of Hilbert's 10^{th} Problem:
  - Use joins of unary relations to encode monomials (products of variables).
  - Use unions to encode sums of monomials.

Unions of Conjunctive Queries

Example: Consider the polynomial $3x_1^4x_2x_3 + 2x_2x_3$

- The monomial $x_1^4x_2x_3$ is encoded by the conjunctive query $P_1(w),P_1(w),P_1(w), P_1(w), P_2(w),P_3(w)$.
- The monomial x_2x_3 is encoded by the conjunctive query $P_2(w),P_3(w)$.
- The polynomial $3x_1^4x_2x_3 + 2x_2x_3$ is encoded by the union having:
  - three copies of $P_1(w),P_1(w),P_1(w), P_1(w), P_2(w),P_3(w)$ and
  - two copies of $P_2(w),P_3(w)$.

Complexity of Query Containment

Class of Queries	Complexity – Set Semantics	Complexity – Bag Semantics
Conjunctive queries	NP-complete CM – 1977	
Unions of conj. queries	NP-complete SY - 1980	Undecidable IR - 1995
Conj. queries with $\neq$, $\leq$, $\geq$	Π_2^p-complete vdM - 1992	
First-order (SQL) queries	Undecidable Gödel - 1931	Undecidable

Conjunctive Queries with ≠

Theorem: Jayram, K …, Vee – 2006
Under bag semantics, the containment problem for conjunctive queries with ≠ is **undecidable**.

In fact, this problem is **undecidable** even if

- the queries use only a single relation of arity 2;
- the number of inequalities in the queries is at most some fixed (albeit huge) constant.

Conjunctive Queries with ≠

Proof Idea:

Reduction from a variant of Hilbert's10th Problem:

Given homogeneous polynomials

$P_1(x_1,\ldots,x_{59})$ and $P_2(x_1,\ldots,x_{59})$

both with integer coefficients and both of degree 5,

is $P_1(x_1,\ldots,x_{59}) \leq (x_1)^5 P_2(x_1,\ldots,x_{59})$,

for all integers $x_1,\ldots,x_{59}$?

Proof Idea (continued)

- Given polynomials P_1 and P_2
  - Both with integer coefficients
  - Both homogeneous, degree 5
  - Both with at most $n=59$ variables
- We want to find Q_1 and Q_2 such that
  - Q_1 and Q_2 are conjunctive queries with inequalities $\neq$
  - $P_1(x_1,\ldots, x_{59}) \leq (x_1)^5 P_2(x_1,\ldots, x_{59})$

    for all integers $x_1, \ldots, x_{59}$

    if and only if

    $Q_1(D) \subseteq_{BAG} Q_2(D)$ for all (bag) databases D.

Proof Outline:

Proof is carried out in three steps.

Step 1: Only consider DBs of a special form.
Show how to use conjunctive queries to encode polynomials and reduce Hilbert's 10th Problem to conjunctive query containment over databases of special form (**no** inequalities are used!)

Step 2: Arbitrary databases
Use inequalities $\neq$ in the queries to achieve the following:

- If a database D is of special form, then we are back to the previous case.
- If a database D is not of special form, then $Q_1(D) \subseteq_{BAG} Q_2(D)$.

- **Step 3:** Show that we only need a single relation of arity 2.

Step 1: DBs of a Special Form - Example

- Encode a homogeneous, 2-variable, degree 2 polynomial in which all coefficients are 1.

  $$P(x_1,x_2) = x_1^2 + x_1x_2 + x_2^2$$

- DBs of special form:
  - Ternary relation TERM consisting of
    - (X_1,X_1,T_1), (X_1,X_2,T_2), (X_2,X_2,T_3)

    all special DBs have precisely this table for TERM
  - Binary relation VALUE
    - Table for VALUE varies to encode different values for the variables x_1, x_2.
- Query Q :- $TERM(u_1,u_2,t)$, $VALUE(u_1,v_1)$, $VALUE(u_2,v_2)$

Step 1: DBs of a Special Form - Example

- $P(x_1,x_2) = x_1^2 + x_1x_2 + x_2^2$
  $x_1 = 3$, $x_2 = 2$, $P(3,2) = 3^2 + 3 \cdot 2 + 2^2 = 19$.
- Query Q :- $TERM(u_1,u_2,t), VALUE(u_1,v_1), VALUE(u_2,v_2)$
- DB D of special form:
  - TERM: $(X_1,X_1,T_1), (X_1,X_2,T_2), (X_2,X_2,T_3)$
  - VALUE: $(X_1,1), (X_1,2), (X_1,3)$
    $(X_2,1), (X_2,2)$

Claim: $P(3,2) = 19 = Q^{BAG}(D)$

Step 1: DBs of a Special Form - Example

- $P(3,2) = 3^2 + 3 \cdot 2 + 2^2 = 19$.
- Query Q :- $TERM(u_1,u_2,t)$, $VALUE(u_1,v_1)$, $VALUE(u_2,v_2)$
- D has TERM: (X_1,X_1,T_1), (X_1,X_2,T_2), (X_2,X_2,T_3)
  VALUE: $(X_1,1)$, $(X_1,2)$, $(X_1,3)$, $(X_2,1)$, $(X_2,2)$
- $Q^{BAG}(D) = 19$, because:
  - $t \to T_1$, $u_1 \to X_1$, $u_2 \to X_1$. Hence:
    $v_1 \to 1,2$, or 3 and $v_2 \to 1$ or 2, so we get 3^2 witnesses.
  - $t \to T_2$, $u_1 \to X_1$, $u_2 \to X_2$. Hence:
    $v_1 \to 1,2$, or 3 and $v_2 \to 1$ or 2, so we get $3 \cdot 2$ witnesses.
  - $t \to T_3$, $u_1 \to X_2$, $u_2 \to X_2$. Hence:
    $v_1 \to 1$ or 2, and $v_2 \to 1$ or 2, so we get 2^2 witnesses.

Step 1: Complete Argument and Wrap-up

- Previous technique only works if all coefficients are 1
- For the complete argument:
  - add a fixed table for every term to the DB;
  - encode coefficients in the query;
  - only table for VALUE can vary.
- **Summary:**
  - If the database has a special form, then we can encode separately homogeneous polynomials P_1 and P_2 by conjunctive queries Q_1 and Q_2.
  - By varying table for VALUE, we vary the variable values.
  - **No** $\neq$-constraints are used in this encoding; hence, conjunctive query containment is **undecidable**, if restricted to databases of the special form.

Step 2: Arbitrary Databases

Idea:

Use inequalities $\neq$ in the queries
to achieve the following:

- If a database D is of special form, then we are back to the previous case.
- If a database D is not of special form, then $Q_1(D) \subseteq_{BAG} Q_2(D)$ necessarily.

Step 2: Arbitrary Databases - Hint

1. Ensure that certain "facts" in special-form DBs appear (else neither query is satisfied).

- This is done by adding a part of the canonical query of special-form DBs as subgoals to each encoding query.

2. Modify special-form DBs by adding gadget tuples to TERM and to VALUE.

- TERM: (X_1,X_1,T_1), (X_1,X_2,T_2), (X_2,X_2,T_3), (T_0,T_0,T_0)
- VALUE: $(X_1,1)$, $(X_1,2)$, $(X_1,3)$, $(X_2,1)$, $(X_2,2)$, (T_0,T_0)

3. Add extra subgoals to Q_2, so that if D is not of special form, then Q_2 "benefits" more than Q_1 and, as a result, $Q_1(D) \subseteq_{BAG} Q_2(D)$.

Step 2: Arbitrary Databases - Example

- $P_1(x_1,x_2) = x_1^2 + x_1x_2 + x_2^2$
- $Poly_1(u_1,u_2,t)$:- $TERM(u_1,u_2,t)$, $VALUE(u_1,v_1)$, $VALUE(u_2,v_2)$
  the query encoding P_1 on special-form DBs.
  - TERM: (X_1,X_1,T_1), (X_1,X_2,T_2), (X_2,X_2,T_3), (T_0,T_0,T_0)
  - VALUE: $(X_1,1)$, $(X_1,2)$, $(X_1,3)$, $(X_2,1)$, $(X_2,2)$, (T_0, T_0)

- Q_1 :- $Poly_1(u_1,u_2,t)$
- Q_2 :- $Poly_2(u_1, u_2, t)$, $Poly_1(w_1, w_2, w)$, $w \neq T_1$, $w \neq T_2$, $w \neq T_3$

Fact:

- If DB is of special form, then Q_2 gets no advantage, because $w \rightarrow T_0$, $w_1 \rightarrow T_0$, $w_2 \rightarrow T_0$ is the only possible assignment.
- If DB not of special form, say it has an extra fact (X_2,X_1,T'), then both Q_1 and Q_2 can use it equally.

Step 2: Arbitrary Databases – Wrap-up

- Additional tricks are needed for the full construction.

- Full construction uses seven different control gadgets.
  - Additional complications when we encode coefficients.
  - Inequalities ≠ are used in both queries.

- Number of inequalities ≠ depends on size of special-form DBs, not counting the facts in VALUE table.
  - Hence, depends on degree of polynomials, # of variables.
  - It is a huge constant (about 59^{10}).

Complexity of Query Containment

Class of Queries	Complexity – Set Semantics	Complexity – Bag Semantics
Conjunctive queries	NP-complete CM – 1977	**Open**
Unions of conj. queries	NP-complete SY - 1980	Undecidable IR - 1995
Conj. queries with $\neq$, $\leq$, $\geq$	Π_2^p-complete vdM - 1992	Undecidable JKV - 2006
First-order (SQL) queries	Undecidable Trakhtenbrot - 1949	Undecidable

Subsequent Developments

- Some progress has been made towards identifying special classes of conjunctive queries for which the containment problem under bag semantics is decidable.

  - Afrati, Damigos, Gergatsoulis – 2010
    - Projection-free conjunctive queries.

  - Kopparty and Rossman – 2011
    - A large class of boolean conjunctive queries on graphs.

The Containment Problem for Boolean Queries

- **Note:**
  For boolean conjunctive queries, the containment problem under bag semantics is equivalent to the Homomorphism Domination Problem.

- **The Homomorphism Domination Problem for graphs**
  Given two graphs G and H, is it true that
  # Hom(G,T) $\leq$ # Hom(H,T), for every graph T?
  (where,
  - # Hom(G,T) = number of homomorphisms from G to T
  - # Hom(H,T) = number of homomorphisms from H to T.

The Homomorphism Domination Problem

Theorem: Kopparty and Rossman -2011

- There is an algorithm to decide, given a series-parallel graph G and a chordal graph H, whether or not # Hom(G,T) $\leq$ # Hom(H,T), for all directed graphs T.

Equivalently,

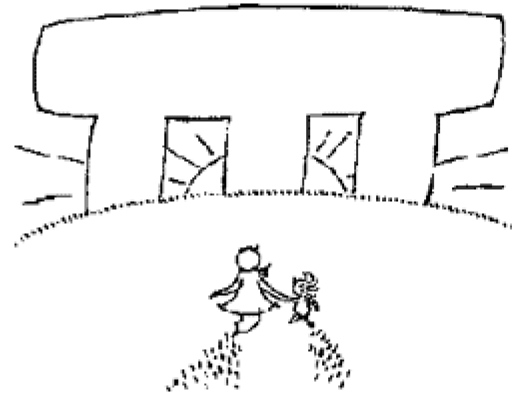
- The conjunctive query containment problem $Q_1 \subseteq_{BAG} Q_2$ is decidable for boolean conjunctive queries Q_1 and Q_2 such that the canonical database $D[Q_1]$ is a series-parallel graph and the canonical database $D[Q_2]$ is a chordal graph.

Note:

Sophisticated proof using entropy and linear programming.

Concluding Remarks

- Twenty years after it was first raised and in spite of considerable efforts, the containment problem for conjuctive queries under bag semantics remains **open**.
- Let us hope that this problem will be settled some time in the next … twenty years.
- But let us also recall another piece of wisdom by Piet Hein.



T.T.T.

Put up in a place
where it is easy to see
the cryptic admonishment
T.T.T.

When you feel how depressingly
slowly you climb
it's well to remember that
Things Take Time.

in: *Grooks* by Peter Hein